A metric for dynamic routing based on variational principles

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Abstract. In this paper, variational principles from theoretical physics are used to describe the process of routing in computer networks as an alternate approach to the traditional graph theory principles. The total traffic currently served on all hops of the route has been chosen as the quantity to minimize. A universal metric has been found for dynamic routing, taking into account the packet loss effect. An attempt was made to derive the metrics of the most popular dynamic routing protocols such as RIP, OSPF, EIGRP from the universal metric.

Keywords. universal metric for dynamic routing, variational principles for routing, intra-domain routing protocols

1. Introduction

In today's networks different routing principles are used, the most time-consuming method of determining the route is the static method. The global network has a large and constantly growing number of routing nodes, so you can not restrict yourself to static routing. A route search in today's networks is performed dynamically. In order to calculate it, different algorithms (Dijkstra's algorithm [1,2], a distance-vector algorithm [3,4], and optimized link state routing [5,6]) are used, forming the basis of routing protocols such as adaptive RIP, OSPF, EIGRP and dynamic BGP.

Each routing protocol, whether it is an intra-domain protocol RIP, OSPF, EIGRP, or an inter-domain protocol BGP [7,8], is based on the appropriate routing metric. But the protocol specification [9] also describes a method of metric measurement, the frequency of refreshing, how to take into account the link-states, etc. This paper only treats the principles of determination of the metric functions, and will not treat the other aspects of routing.

Despite the large number of existing algorithms, the question of constructing a universal metric has not yet been resolved in a general form. An analytical solution for the description of packet forwarding (routing) has not been found. Currently, the process of route selection is to determine the appropriate metric for characterizing the features of the route. Subsequently, its values for the different routes are compared, and the route with the minimum value is chosen.

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Here we try to construct a universal metric for dynamic routing based on an extremal principle. Fundamentally new mathematical tools make up the foundation of our approach: variational methods replace standard graph theory [10]. This principle allows of representing the general form of the metric as a sum of positive values for all hops of the route. Thus, we can justify the application of graph theory to solve routing problems.

The aim of this work is to find a universal metric for dynamic routing based on a variational principle. Different areas of physics have long used the concept of extremum for specially selected quantities:

- In classical mechanics, this is called the principle of least action.
- Similarly, extremal principles have been formulated in classical electrodynamics and the general theory of relativity, with the only difference being that the action of the particles in the external fields is now added to the action, describing the variation of the fields themselves. In quantum electrodynamics and quantum gravity, these principles also follow the corresponding functional integrals.
- In geometric optics, this role is played by Fermat's principle (the principle of least time).
- In thermodynamics, the quantity for which the extremum is sought is the entropy.

In theoretical physics, all fundamental laws for all sections can be derived on the basis of an extremum principle. This statement is perfectly illustrated in the course of theoretical physics [11].

The present article is organized as follows: the second part explains the choice of the metric function as a function of the action similar to Fermat's principle in optics, which minimizes the optical path length. Furthermore, in Section 3.1 we derive a metric dynamic protocol RIP as the simplest limiting case of the metric found. Section 3.2 shows that a simplification of the metric function for intra-routing data channels of minimum length coincides with the metric OSPF up to some constants. Section 3.3 analyzes the it EIGRP metric function and the coordination unit quantities used in the metric function. Last, some conclusions are drawn and future lines of research are outlined.

2. Selection of the action

As a fundamental principle, Fermat's principle was selected as the best for dynamically searching for a network route. Fermat's principle in geometric optics is a postulate, directing the ray of light to move from a starting point to an end point along the path that minimizes the optical path length

$$\delta \int_A^B n dl = 0, \tag{1}$$

or for areas with a constant refractive index,

$$S = \min \sum_i n_i l_i, \tag{2}$$

where l_i is the distance travelled by the light in a medium with a refractive index of n_i.

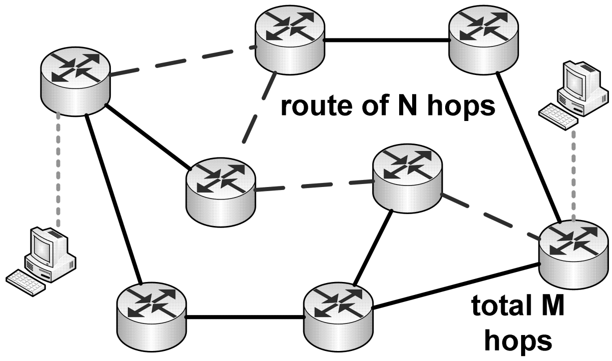

Figure 1. Routing scheme

This principle is chosen as the starting point for describing routing in computer networks.

Traffic that is currently served by the route has been selected as an action. the traffic is found using Little's Law from queuing theory. The classical formulation of this theorem states that the average number of customers N in a stable functioning system is the product of the average arrival rate of customers λ with the average time T they spend in the system,

$$N = \lambda T. \tag{3}$$

In this paper, the notation for the basic characteristics of the network is adopted from the [12,13,14]. Suppose we are given a network consisting of M routers connected to communication channels (see Fig. 1), then an arbitrary route consists of N hops. The following definitions are used:

- A hop is a "point to point" connection which includes a router and a data channel that connects the router to another one.
- The route is the set of hops through which packets are sent from the sender to the recipient.

An arbitrary hop is characterized by the values (see Fig. 2)

- The capacity C_i is the maximum IP-layer throughput that the path can provide to a flow, when there is no competing traffic load.
- The available bandwidth B_i is the maximum IP-layer throughput that the path can provide to a flow, given the path's current cross traffic load.
- D_i, the delay, is the time it takes for the packet to be transmitted over the hop.
- p_i, the percentage of losses, is the percentage of lost packets as a fraction of the total number of transmitted packets.
- D_i^{buf}, the duration of the dejitter buffer, is the period of time which is necessary for saving the packets on the receiving side of the router in order to eliminate the effect of irregularity on delivery of the package. This is the time between

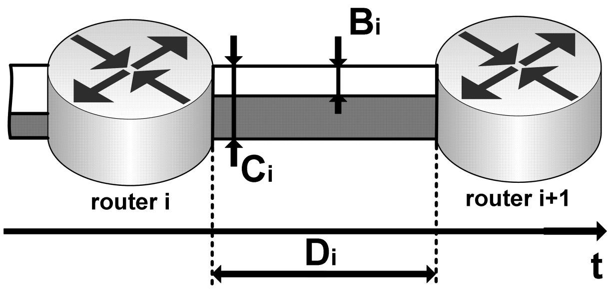

Figure 2. Illustration of basic quantities

the maximum allowable waiting time in the queues and the average value of the delays. The network jitter j is defined as the variation of the delay.

Little's law allows finding the traffic that is currently served by the system on the testing route in Fig. 1:

$$F = \sum_{i=1}^{N} (C_i - B_i) D_i, \tag{4}$$

where the summation is over all parts of the route.

However, this function does not take into account the impact of the packet loss p_i in the network hop. In order to describe this effect, we note that the packet is considered lost if the time of delivery exceeds the magnitude D^{max}. Therefore, the metric function of Eqn. (4) should introduce an additional term

$$F = \sum_{i=1}^{N} (C_i - B_i)((1 - p_i) D_i + p_i D^{max}), \tag{5}$$

This equation can be reduced to

$$F = \sum_{i=1}^{N} (C_i - B_i)(D_i + p_i D_i^{buf}), \tag{6}$$

where $D_i^{buf} = D^{max} - D_i$ is the time to allow for smoothing the effect of the delay variation (network jitter) [15]. Let us try to estimate the dependence of the dejitter buffer D_i^{buf} on the value of the jitter, j. For this assessment, we must estimate the percentage of packet loss due to the delay variation. The corresponding expression can be derived from a Poisson distribution, or from knowing the type of delay distribution [16]. Both these approaches will lead to the expression

$$p = e^{-\lambda D^{buf}}, \tag{7}$$

where $\lambda = 1/j$. For the dejitter buffer we obtain

$$D^{buf} = -j \ln p, \tag{8}$$

for a more specific routing problem, $D^{buf} = 7j$ can be used. This estimate is based on the assumption that the overall percentage of packet loss is less than 0.5% [17] for good traffic, and only 20% of the data loss is caused by the dejitter effect [18].

The function from Eqn. (6) can be chosen as an analogue of the action used in physics. The minimum condition for this quantity allows us to select the best route,

$$\min_{j=\overline{1,K}} F_j = \min_{j=\overline{1,K}} \sum_{i=1}^{N} (C_i - B_i)(D_i + p_i D_i^{buf}), \tag{9}$$

where K is the number of comparable routes.

In the next section, an attempt will be made to derive the basic metric of the commonly used routing protocols from this universal metric.

3. Metrics of the major routing protocols

3.1. RIP

With the explosive growth of the global network, engineers needed to quickly create working routing algorithms. Therefore, initially a simple algorithm was created, which later became more sophisticated. Thus there is one additional condition: new algorithms should be based on the old metrics in limiting cases. So these metric functions often only contain the linear terms of the complex dependencies of routing, which have been described in Eqn. (9). In this and subsequent sections, the main function used in the routing protocols will be obtained by simplifying the universal metric.

The simplest dynamic routing protocol is the “Routing Information Protocol” (RIP). The number of hops is used as the metric. Fig. 3 illustrates how the RIP protocol chooses a route: instead of three more sections of the high-speed backbone, RIP selects the best route in terms of the number of hops, even if this is much worse in terms of speed.

For the universal routing metric from Eqn. (6), $B_i = B$, $C_i = C$, $D_i = D$, $p_i = 0$, then

$$F = (C - B)DN, \tag{10}$$

where N is the number of hops, and the minimization procedure leads to the choice of the route with the minimum number of hops.

3.2. OSPF

Open Shortest Path First (OSPF) is a dynamic routing protocol based on link-state technology that uses Dijkstra’s algorithm for finding the shortest path.

OSPF allows for optimum bandwidth utilization. The metric determines the weight of the edge of the graph to be $10^8/B_i$, where B_i is in bits/s. The total metric function $W_j = \sum_{i=1}^{N} 10^8/B_i$ is constructed for the comparison of the K possible routes, and then the route with the lowest value W_j of the metric is chosen

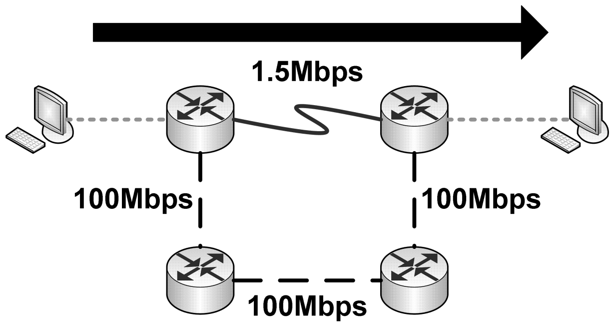

Figure 3. Illustration of routing for RIP metrics

$$F = \min_{j=\overline{1,K}} \sum_{i=1}^{N} \frac{10^8}{B_i}. \quad (11)$$

The OSPF metric can be compared with the metric found in Section 2 of this article, on the separate hops of the route. For this analysis, we use the simplified expression for the metric function of Eqn. (4). In this equation, the fixed packet delay in the transmission under a separate hop can be represented as the sum of two terms [16]

$$D_i = \frac{l_i}{c_o} + \frac{W}{B_i}, \quad (12)$$

where l_i is the link length, $c_o \approx 0.6c$ is the speed of light in the fiber optic medium, W is the size of the transmitted data packet, bounded above by MTU (Maximum Transmission Unit, 1500 bytes). The first term describes the propagation of the signal, and the second term is responsible for the packet processing in the communication channel. For practical calculations, the packet size can be estimated as 10^4 bits. Here we refer to fixed delay as the minimum path transit time for the given packet size W.

Intra-domain routing is characterized by short lengths of data links. Therefore, in Eqn. (12), one should take $l_i = 0$, and the metric function without packet loss is

$$F_i^{th} = \frac{C_i W}{B_i} + W, \quad (13)$$

because the available bandwidth is always less than the channel capacity, and so the metric function of Eqn. (13) is always positive. Up to a constant, it coincides with the metric OSPF.

However, consideration of the length of the data link allows determining the limits of applicability of the metric OSPF, as well as ways to further upgrade the metric function. In this case, a universal metric function for single hops can be written as

$$F_i^{th} = \frac{C_i l_i}{c_o} - W - \frac{l_i}{c_o} W + \frac{W C_i}{B_i}. \quad (14)$$

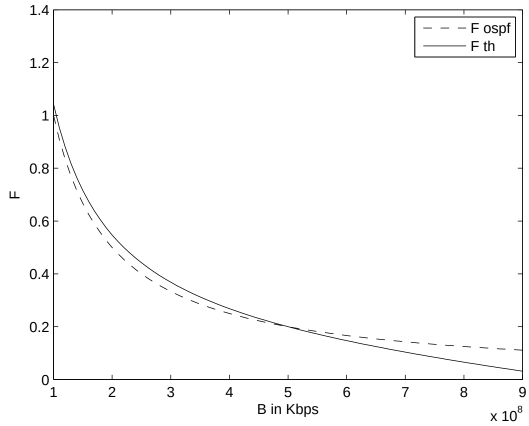

Figure 4. Comparison of the metric functions

The first three terms of Eqn. (14) are affine functions in the variable B_i. The theoretical metric F_i^{th} of Eqn. (14) and the metric OSPF F_i^{OSPF} coincide up to a constant when the available bandwidth is equal to

$$B_i^0 = C_i - \frac{c_o W}{l_i}. \tag{15}$$

In order to understand how different are these two metrics, we will determine the absolute error ΔF for the value $B_i = B_i^0 - kC_i$, where k is a small constant factor.

$$\Delta F_i = F_i^{th} - F_i^{OSPF} = kC_i \left(\frac{l_i}{c_o} + \frac{WC_i}{B_i^2} \right) \tag{16}$$

Figure 4 compares the two metric functions for a data channel of length $l = 4$ km, channel capacity $C = 1$ Gbps, and with the available bandwidth varying from ten to ninety percent of the channel capacity. The packet size is assumed to be 10^4 bits.

The inverse association with a relatively short length of the channel ($l_i \ll cW/B_i$) and without packet loss describes the process of dynamic routing as good enough.

3.3. EIGRP

There are some differences between EIGRP and OSPF. First, this metric is assigned to the whole route, it can not be calculated for a single hop. In addition, the OSPF metric is different from the theoretical function. While summing it over the route, this error can accumulate, especially when the channel condition $l_i \ll cW/B_i$ is not satisfied. The OSPF metric also does not account for packet loss on the route.

The EIGRP metric employs five basic variables (the default is only two)

- $bandwidth = \frac{10^7}{B} 256$, where $B = \min B_i$ in Kbps is the smallest available bandwidth between the two points investigated (the default);
- $delay = D * 256$, D is the total delay along the route ($D = \sum_{i=1}^{N} D_i$) in tens of microseconds;

- $relability = 256 * p$ is the worst reliability score along the route and is dependent on the packet loss, in our notation $p = \max p_i$;
- $loading = 256 * l$ is the channel utilization rate, where $l = (C_i - B_i)/C_i$;
- MTU is the smallest Maximum Transmission Unit (MTU) along the path.

By default, the variables *bandwidth* and *delay* are used to calculate the metric with some weight coefficients. The remaining metric functions are not recommended, as this will result in frequent route recalculation. EIGRP is a metric using two types of coefficient, the weight and the base. By default, the basic values of the coefficients are $K1 = K3 = 1$, $K2 = K4 = K5 = 0$.

If the values of the coefficients are equal to the basic default values, taking into account the weighting coefficients, the metric function will be

$$W = bandwidth + delay. \tag{17}$$

For inter-domain routing, the length of the data transmission channels l_i in Eqn. (12) can not be ignored. In this case, the first term on the left-hand side of Eqn. (12) becomes dominant. In the summation over all route hops, the packet delay on route D can be expressed as

$$D = \sum_{i=1}^{N} D_i = \frac{\sum_{i=1}^{N} l_i}{c_o} + \frac{NW}{B}. \tag{18}$$

The first term is equal to the total length of the route to the speed of propagation of the signal, and the second term can be assessed from the top $\sum_{i=1}^{N} \frac{W}{B_i} \leq \frac{NW}{\min_{i=\overline{1,N}} B_i} = \frac{NW}{B}$

Eqn. (16) indicates that the metric OSPF generally must be supplemented by a term proportional to the packet delay on the route D. This can be done when the contribution of the first term will be dominant. The presence of kC in Eqn. (16) indicates that the variables *bandwidth* and *delay* in the EIGRP metric of Eqn. (17) should be included with different weights.

We can make the following estimate for the contribution of the terms in the right-hand side of Eqns. (16) and (18). Denote by e_1 the ratio between the terms of Eqn. (18), so for the available channel bandwidth, we obtain

$$B = \frac{c_o e_1 W}{l}, \tag{19}$$

i.e., for the length of the data link to the 1000 km contribution of the second term will be less than 10% if the channel width is greater than 160 Kbps. Therefore, the contribution of the second term in the total metric EIGRP when inter-domain routing can be considered minimal one.

At the same time, the presence of the second term in Eqn. (18) leads to the determination of the weight ratio of the metric EIGRP. To do this, instead of the coefficient kC, they change the unit of measure packet delay. In Eqn. (17), the first term is proportional to $\frac{10^7}{B}$, while the second term contains the term $\frac{10^5}{B}$ for $N \approx 10$. In order to bring the two terms to a single scale for $B = 10^{10}$ bps, it is necessary to introduce an additional factor of 10^{-5}. In practice this means that the delay in the EIGRP metric (17) is measured in tens of microseconds, not in seconds.

Reproducing the dependence found in the universal metric (9) will require the introduction of many weights. The most complete EIGRP metric contains all the same network variables, which are used in the universal metric (9), and the selection of weights can properly compare the EIGRP metric in a selected range of values.

4. Summary and future research

The main focus of this paper was to use the mathematical apparatus of theoretical physics to describe the process of routing in computer networks. Information theory based on Claude Shannon [19,20] also relies on the introduction of the concept of entropy. The problem of determining a route is similar to determining a trajectory of motion, and in problems of this type a variational principle is always used. Little's law of queuing theory gave a hint about the type of variable which it is necessary to minimize. The traffic currently served by the compared network route was selected as such a variable. The formula obtained has been improved with the consideration of the effect of packet loss on each network hop.

Vast experience in organizing dynamic routing has been gained in the operation of a global network with an extremely complicated topology, which was obtained from the most popular algorithms such as RIP, OSPF, and EIGRP. In this paper we attempted to derive the metric functions of these algorithms, based on the universal metric determined. All the variables that are contained in the universal metric of Eqn. (9) were used in constructing these algorithms. The experiments helped establish the basic types of dependency, and the metric was constructed by adding terms with weighting factors to previously established terms.

The question of the practical implementation of the metric and the construction of a new routing algorithm are not considered in this work because this would require separate pilot studies in order to calculate the universal metric with the lowest cost, without loading the network measurements, and allowing the route to change frequently.

We should also consider further theoretical studies, since the principle of extrema is inextricably linked with variational principles. The main thing to understand is how to produce a variation of the variables, and which effect will be used to describe the resulting equation. It is likely that this will describe the features of a single router with multiple channels, in contrast to the universal metric function, which describes the extended route.

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